Credal Networks for Operational Risk Measurement and Management ⋆

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Abstract. According to widely accepted guidelines for self-regulation, the capital requirements of a bank should relate to the level of risk with respect to three different categories. Among them, *operational risk* is the more difficult to assess, as it requires merging expert judgments and quantitative information about the functional structure of the bank. A number of approaches to the evaluation of operational risk based on Bayesian networks have been recently considered. In this paper, we propose *credal networks*, which are a generalization of Bayesian networks to imprecise probabilities, as a more appropriate framework for the *measurement* and *management* of operational risk. The reason is the higher flexibility provided by credal networks compared to Bayesian networks in the quantification of the probabilities underlying the model: this makes it possible to represent human expertise required for these evaluations in a credible and robust way. We use a real-world application to demonstrate these features and to show how to measure operational risk by means of algorithms for inference over credal nets. This is shown to be possible, also in the case when the observation of some factor is vague.

Key words: Credal networks, Operational Risk, Imprecise probabilities

1 Introduction

In the last two decades, the definition of standards for the stability of the international banking system, aiming at improving consistency of international capital regulations, has been unanimously recognized by the banking institutes worldwide as a keypoint for their self-regulation. The *Basel Committee on Banking Supervision*, which is the main authority in this field, defines the guidelines, accepted by national supervisory institutes worldwide, to measure the capital adequacy of each bank and to guarantee it to undergo a minimum standard. Accordingly, three different categories of risk, associated respectively to market, credit and operational activities, and the corresponding minimal capital

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requirements hedging each category, should be estimated. While the evaluation of market and credit risk is traditionally supported by (large amounts of) historical data and well-established measurement methods, the evaluation of the latter category, which is called *operational risk* (Sect. 2), is more difficult, requiring the development of models that combine expert opinions with quantitative information. Yet, a standard model was not explicitly specified by the Basel Committee.

To fill this gap, some scholars have proposed *Bayesian networks* (Sect. 3.1) as a possible framework to measure operational risk with respect to the Committee requirements. Bayesian networks are graphical models, whose quantification requires a precise elicitation of the probabilistic relations among the different factors. Yet, this requirement clashes with the kind of uncertainty characterizing qualitative expert judgments about operational risk.

For this reason, we regard *credal networks* (Sect. 3.2), which are a generalization of Bayesian networks to imprecise probabilities, as a more credible model for operational risk. Credal networks allow for the specification of intervals (or, more generally, closed convex sets of mass functions) instead of single values of probability: this appears to be better suited to capture human knowledge. This flexibility regards also the observation of the variables: credal nets can cope with vague observations, where a condition of partial or complete ignorance about the actual state of an observed variable holds (Sect. 3.3).

To demonstrate and clarify these key features, a Bayesian network for operational risk evaluation is considered (Sect. 4.1). This model is first transformed into a credal network by more reliable probabilistic assessments (Sect. 4.2). We then show, by this example, how to employ credal networks for both *measurement* and *management* of operational risk. To solve the first problem, the network is fed with the available evidential information about the bank under consideration. The posterior probability intervals for the risk level can be therefore computed by standard algorithms for credal networks (Sect. 5). These tests are considered also in presence of *soft evidence*, that means observation unable to reveal the state of a variable, which is therefore modeled by (imprecise) probabilistic assessments. A simulation of a management problem, consisting in the evaluation of the configuration of the variables minimizing operational risk, is also considered (Sect. 6).

Overall, we regard this work as a first step towards a systematic approach to operational risk measurements based on credal networks. Conclusions and possible developments in this direction are reported in Sect. 7.

2 Operational Risk

In June 2004, the Basel Committee on Banking Supervision (simply called the *Committee* in the following) released an important document known as *Basel II accord* [9]. This document defines a comprehensive measure and a minimum standard for capital adequacy, which has been implemented by national supervisory authorities worldwide. The aim of these guidelines is to contribute to the stability of the international banking system, by improving the consistency of capital regulations, making regulatory capital more risk sensitive, and promoting enhanced risk-management practices among financial institutions. Basel II is a revised version of a first accord (*Basel I*), that has been employed since 1988 [7].

Basel II sets standards for measurement and minimal capital requirements with respect to three different types of risk that can be incurred by financial institutions: market, credit, and operational risk. While market risk and credit risk, that were already considered in Basel I, can be evaluated by standard techniques, the evaluation of the *operational risk* (OR), that appears for the first time in Basel II as a separate category of risk, is more difficult.

OR is defined as *"the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events"* [9]. The definition of OR is very general and encompasses several possible events like for example internal fraud, business disruption and system or delivery failures [8]. It is important to note that, unlike factors driving market and credit risk, factors driving OR are mainly internal and specific to the single financial institutions. This feature, together with the scarcity of reliable historical data regarding losses due to OR events, is the main reason for the lack of a standard method in this field.

The Committee has proposed three possible approaches for measuring OR and calculating the corresponding minimal capital requirement: (i) the *Basic Indicator Approach*, (ii) the *Standardized Approach*, and (iii) the *Advanced Measurement Approach* (AMA). Of these approaches, only the latter is risk sensitive, while the others adopt the conservative choice to set the minimal capital requirements as a fixed percentage of the the gross annual income in different business lines, and are therefore useless for OR management. In the AMA, the calculation of minimal capital requirement is based on OR measurement systems developed by the bank itself. Basel II is not prescriptive with respect to the particular models that should be used by financial institutions following the AMA to measure OR, but describes several qualitative and quantitative criteria that should be met by the measurement model in order to be appropriate, e.g., *"use of internal data, external relevant data, scenario analysis and factors reflecting the business environment"* [9]. To meet these requirements, it is therefore necessary to apply models merging expert opinions with data. In Sect. 4, we propose *credal networks* (which are defined in Sect. 3.2) as an appropriate mathematical framework for these evaluations.

3 Mathematical Aspects

In this section we review the basics of Bayesian nets and their extension to *imprecise probabilities* [12], i.e., credal nets. Both the models are based on a collection of random variables $\mathbf{X} \equiv (X_1, \ldots, X_n)$, which take values in finite sets, and a directed acyclic graph $\mathcal{G}$, whose nodes are in one-to-one relationship with the variables of $\mathbf{X}$. For both models, we assume the *Markov condition* to make $\mathcal{G}$ represent probabilistic independence relations between the variables in $\mathbf{X}$: every variable is independent of its non-descendant non-parents conditional on its parents. What makes Bayesian and credal nets different, is a different notion of independence and a different characterization of the conditional mass functions for each variable given the possible values of the parents, to be detailed next.

Regarding notation, for each $X_i \in \mathbf{X}$, Ω_{X_i} is the possibility space of X_i, x_i a generic element of Ω_{X_i}, $P(X_i)$ a mass function for X_i and $P(x_i)$ the probability that $X_i = x_i$. A similar notation with uppercase subscripts (e.g., X_E) denotes arrays (and sets) of variables in $\mathbf{X}$. The parents of X_i, according to $\mathcal{G}$, are denoted by Π_i and for each $\pi_i \in \Omega_{\Pi_i}$, $P(X_i|\pi_i)$ is the conditional mass function for X_i given the joint value π_i of the parents of X_i.

3.1 Bayesian Networks

In the case of Bayesian networks [10], the modeling phase involves specifying a conditional mass function $P(X_i|\pi_i)$ for each $X_i \in \mathbf{X}$ and $\pi_i \in \Omega_{\Pi_i}$; and the standard notion of probabilistic independence is assumed in the Markov condition. A Bayesian net can therefore be regarded as a joint probability mass function over $\mathbf{X}$ that factorizes as follows: $P(\mathbf{x}) = \prod_{i=1}^n P(x_i|\pi_i)$, for each $\mathbf{x} \in \Omega_{\mathbf{X}}$, because of the Markov condition.

A Bayesian network can be regarded as an expert system, from which we can extract probabilistic information about its variables. The updated belief about a queried variable X_q, given some evidence $X_E = x_E$, is given by Bayes rule:

$$P(x_q|x_E) = \frac{P(x_q, x_E)}{P(x_E)} = \frac{\sum_{x_M} \prod_{i=1}^n P(x_i|\pi_i)}{\sum_{x_M, x_q} \prod_{i=1}^n P(x_i|\pi_i)}, \tag{1}$$

where $X_M \equiv \mathbf{X} \setminus (\{X_q\} \cup X_E)$, the domains of the arguments of the sums are left implicit, and the values x_i and π_i are consistent with (x_q, x_M, x_E). Despite its NP-hardness in general, the problem of computing (1) can be efficiently solved for Bayesian networks defined over singly-connected graphs with standard propagation schemes [10]. Similar techniques apply also for general topologies with increased computational time.

3.2 Credal Networks

Credal nets relax Bayesian nets by allowing for imprecise probability statements. This generalization is based on the fundamental notion of *credal set* [5]. We define a *credal set* $K(X)$ as the convex hull of a collection of mass functions over X. We assume this collection to be finite. $K(X)$ contains therefore an infinite number of mass functions, but only a finite number of *extreme mass functions*, corresponding to the (geometrical) *vertices* of the convex hull. Inference based on a credal set is equivalent to that based only on its extreme mass functions [12], so that the nature of the problem we face is inherently combinatorial.

The specification of a *credal network* over $\mathbf{X}$ consists in the assessment of a conditional credal set $K(X_i|\pi_i)$ for each $X_i \in \mathbf{X}$ and $\pi_i \in \Omega_{\Pi_i}$. We assume that the credal sets of the net are *separately specified*: this implies that selecting a mass function from a credal set does not influence the possible choices in others.

The *strong extension* $K(\mathbf{X})$ of a credal network [3] is defined as the convex hull of the joint mass functions $P(\mathbf{X})$, which, for each $\mathbf{x} \in \Omega_{\mathbf{X}}$, are such that:

$$P(\mathbf{x}) = \prod_{i=1}^{n} P(x_i|\pi_i), \quad \begin{array}{l} P(X_i|\pi_i) \in K(X_i|\pi_i), \\ \text{for each } X_i \in \mathbf{X}, \pi_i \in \Omega_{\Pi_i}. \end{array} \tag{2}$$

Observe that any extreme mass function $P(\mathbf{X})$ of $K(\mathbf{X})$ can be identified with a Bayesian net. In other words, a credal net is equivalent to a set of so-called *compatible* Bayesian nets. Inference over a credal net is intended as the computation of lower and upper expectations over the strong extension $K(\mathbf{X})$. Updating consists therefore in the computation of the lower and upper bounds of (1), denoted respectively as $\underline{P}(x_q|x_E)$ and $\overline{P}(x_q|x_E)$, evaluated over all the compatible Bayesian nets. Credal nets updating is an NP-hard task [4], for which a number of approximate algorithms have been proposed [3].

3.3 Soft Evidence by Credal Networks

In a recent paper [2], credal nets updating has been generalized to situations where the observations of some variables are missing according to an unknown process. Remarkably, we can reproduce this situation by simply augmenting the network with *auxiliary binary child nodes*, whose (imprecise) quantification reproduces the ignorance about the missingness process. This approach can be naturally extended to various forms of *soft evidence* about a variable. If an observation cannot reveal the actual state of a variable, but the knowledge provided by this observation can be modeled as a credal set, we can still fed the credal net with this (soft) information, by an appropriate quantification of the auxiliary child. Section 5 explains this idea by an application to OR.

4 A Credal Network for Operational Risk Measurement

Several authors have proposed Bayesian nets as a possible modeling framework for OR measurement [1, 6, 11]. These models make it possible to merge qualitative and quantitative factors driving OR measurements in a very natural and easy verifiable way. The most critical and time-consuming task for the realization of a Bayesian net in practical OR measurements is the quantification of the probabilities. This process can be very critical and arbitrary, e.g., what is the probabilistic assessment modeling the fact that the trade type of a bank is more likely to be standard than not? The problem is that these kinds of knowledge cannot be modeled by precise probabilistic statements. On the other hand, a more credible model of such qualitative judgment is the credal set made of *all* the mass functions assigning more than half of the mass to the state *standard*. This is the main reason why we regard credal nets as a model more suited than Bayesian nets for OR measurements. To demonstrate the main features of this approach, a concrete application based on a Bayesian net for OR measurement is considered.

4.1 A Bayesian Network for Operational Risk (Adusei-Poku 2005)

Let us consider the Bayesian network for OR measurement detailed in [1]. As an indicator of the level of OR, the number of SETTLEMENTS FAILURES in foreign exchange operations (e.g., delays, wrong or misdirected payments) in a typical week of a particular bank is considered. For our purposes, the following categorization is sufficient: less than 5 failures (corresponding to *low risk*), from 6 to 10 failures (*medium risk*), more than 10 failures (*high risk*).[1] Thirteen variables, characterizing the internal structure of the bank under consideration, and affecting this number of failures, are indeed considered. Figure 1 depicts the probabilistic dependencies between these variables and the OR indicator (according to the Markov condition). We point the reader to [1, Appendix B] for the quantification of this network.

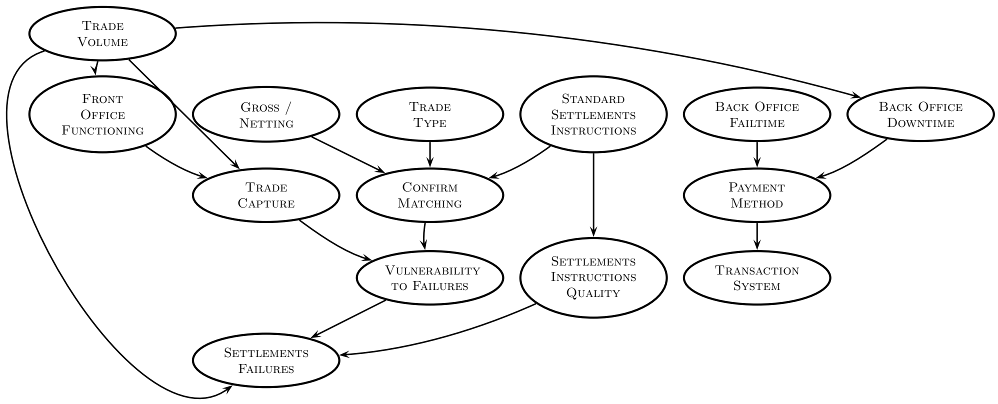

Fig. 1. A directed acyclic graph depicting dependencies between the OR level (represented by the node SETTLEMENTS FAILURES) and its triggering factors.

4.2 From Bayesian to Credal Network

The data about the conditional probabilities for the Bayesian net of Section 4.1 attest one of the main limitations of these models for the representation of human expertise. For some variables, the specification in [1] actually reports ranges instead of precise values for the probabilities assessed by the experts. But in the end, a *typical value* in this range has to be adopted for use with Bayesian nets. Thus, the modeling phase requires artificially strong assessments, that can be avoided using credal nets. To show the benefit, we can therefore transform the Bayesian network into a credal network.

The ranges provided for the probabilities of the two states of the node TRADE TYPE, (90–99% for *standard* and the complement for *non standard*) are reproduced by a credal set with two extreme mass functions, and we similarly proceed

[1] We use SMALL CAPITAL for the variables and *slanted* for the states.

for the nodes Standard Settlements Instructions, Gross/Netting, and Settlements Failures, obtaining a credal net with 216 compatible Bayesian nets. This relatively small number makes it possible to solve inferences exhaustively by simply solving each Bayesian net independently.

5 Operational Risk Measurement

The credal net described in the previous section can be clearly employed to measure OR. To this extent, it is sufficient to collect the available information about the variables affecting the OR, and update the probabilities of the node Settlements Failures. These probabilities for a simulated scenario consisting of a bank where the Front Office Functioning is *proper*, the Settlements Instructions Quality is *average* and the Payment System is *manual* are: 86.69–91.94% for *low*, 7.70–12.87% for *medium*, and 0.35–0.45% for *high risk*. This result is largely acceptable from a financial point of view, and based on much more reliable assumptions than the corresponding result obtained by the Bayesian net (90.01% for *low*, 9.61% for *medium*, and 0.38% for *high*).

According to Sect. 3.3, we can also model a situation where only soft evidence about these variables is available. Assume for instance that the observation of the Front Office Functioning certifies that the system is certainly not *down*, but we cannot decide whether it is *malfunctioning* or *properly functioning*, even if our observation suggests that the latter case is more likely. This is a soft evidence we can model by a credal set. Thus, we fed the net with this information by simply adding a binary child to the Front Office Functioning, assuming two possible specifications of the relative conditional probability table:

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 1 \end{bmatrix} \right\}, \tag{3}$$

where the columns of these tables correspond respectively to the states *down*, *proper*, and *malfunctioning*. The first row of each tables reproduces a vertex of the credal set modeling our soft evidence. As an obvious generalization of the results presented in [2], we conclude that assuming the child node in the state corresponding to the first row of the table, we can compute the OR level of this scenario. The posterior probabilities (85.54–91.94% for *low*, 7.70–14.05% for *medium* and 0.35–0.54% for *high risk*) reveal a relative robustness of the simulated scenario with respect to the partial ignorance about the state of the Front Office Functioning.

6 Operational Risk Management

Finally, let us consider a management problem consisting in the evaluation of the effect of Front Office Functioning, Settlements Instructions Quality, and Payment Method to the level of OR. By simply analyzing the results reported in Tab. 1, we note that the configuration *proper*/*excellent*/*STP* assigns the highest probability to *low risk*. More generally, these kinds of simulations provide important information about the role played by the different variables in the determination of the overall level of OR.

Table 1. Posterior probabilities of OR for 18 simulated scenarios.

Front Office functioning	Quality of SI	Payment System = *STP* P(Failures Settlements) [%]			Payment System = *manual* P(Failures Settlements) [%]		
		low	*medium*	high	*low*	*medium*	*high*
proper	*excellent*	[97.97,98.23]	[01.77,02.02]	[00.00,00.00]	[97.63,97.91]	[02.09,02.36]	[00.00,00.01]
proper	*average*	[88.40,93.36]	[06.38,11.27]	[00.25,00.33]	[86.69,91.94]	[07.70,12.87]	[00.35,00.45]
proper	*bad*	[46.54,54.35]	[42.49,47.58]	[03.15,05.89]	[44.40,52.66]	[43.39,48.76]	[03.95,06.85]
mal func.	*excellent*	[96.65,96.83]	[03.15,03.33]	[00.02,00.02]	[96.06,96.11]	[03.87,03.92]	[00.02,00.02]
mal func.	*average*	[81.35,86.64]	[12.63,17.84]	[00.73,00.81]	[79.33,84.13]	[14.90,19.66]	[00.97,01.01]
mal func.	*bad*	[37.69,46.17]	[46.48,51.96]	[07.31,10.43]	[35.55,43.48]	[47.18,52.94]	[08.75,11.59]
down	*excellent*	[95.61,96.03]	[03.92,04.34]	[00.04,00.05]	[94.86,95.12]	[04.82,05.08]	[00.06,00.06]
down	*average*	[78.67,84.28]	[14.66,20.06]	[01.06,01.26]	[76.46,81.49]	[17.12,22.01]	[01.38,01.53]
down	*bad*	[35.74,44.13]	[46.09,51.00]	[09.67,13.46]	[33.55,41.32]	[47.19,51.86]	[11.36,14.81]

7 Conclusions and Further Work

The main goal of this paper is to point out as credal networks can be regarded as a natural framework for OR measurement problem, with the same desirable features of Bayesian networks, but allowing for more freedom and robustness in the elicitation of the underlying probabilities and in the treatment of the available evidence. This argument is demonstrated by a concrete application, which represents a starting point for similar and more complex applications.

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