Hazard Assessment of Debris Flows by Credal Networks*

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Abstract: Debris flows are destructive natural hazards that affect human life, buildings, and infrastructures. Despite their importance, debris flows are only partially understood, and human expertise still plays a key role for hazard identification. This paper proposes filling the modelling gap by using *credal networks*, an imprecise-probability model. The model uses a directed graph to capture the causal relationships between the triggering factors of debris flows. Quantitative influences are represented by probability intervals, determined from historical data, expert knowledge, and theoretical models. Most importantly, the model joins the empirical and the quantitative modelling levels, in the direction of more credible inferences. The model is evaluated on real case studies related to dangerous areas of the Ticino Canton, southern Switzerland. The case studies highlight the good capabilities of the model: for all the areas the model produces significant probabilities of hazard.

Keywords: Debris flows; credal networks; imprecise Dirichlet model; probability intervals; updating.

1 INTRODUCTION

Debris flows are among the most dangerous and destructive natural hazards that affect human life, buildings, and infrastructures. Starting from the '70s, significant scientific and engineering advances in the understanding of the processes have been achieved (see Costa and Wiekzorek [1987]; Iverson et al. [1997]). Yet, human expertise is still fundamental for hazard identification as many aspects of the whole process are still poorly understood.

This paper presents a *credal network* model of debris flow hazard for the *Ticino canton*, southern Switzerland. *Credal networks* (Cozman [2000]) are imprecise-probability models based on the extension of *Bayesian networks* (Pearl [1988]) to sets of probability mass functions (see Sec. 2.2). *Imprecise probability* is a very general theory of uncertainty developed by Walley [1991] that measures chance and uncertainty without sharp probabilities.[2]

The model represents expert's causal knowledge by a directed graph, connecting the triggering factors for debris flows (Sec. 3.1). Probability intervals are used to quantify uncertainty (Sec. 3.2) on the basis of historical data, expert knowledge, and physical theories. It is worth emphasizing that the credal network model joins human expertise and quantitative knowledge. This seems to be a necessary step for drawing credible conclusions. We are not aware of other approaches with this characteristic.

The model presented here aims at supporting experts in the prediction of dangerous events of debris flow. We have made preliminary experiments in this respect by testing the model on historical cases of debris flows happened in the Ticino canton. The case studies highlight the good capabilities of the model: for all the areas the model produces signifi-

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[2]See Walley [1996b] for a thorough comparison of imprecise probability with other measures of uncertainty popular in artificial intelligence, such as belief functions and possibility measures.

cant probabilities of hazard. We make a critical discussion of the results in Sec. 4, showing how the results are largely acceptable by a domain expert.

2 BACKGROUND

2.1 Debris Flows

Debris flows are composed of a mixture of water and sediment.

Three types of debris flow initiation are relevant: erosion of a channel bed due to intense rainfall, landslide, or destruction of a previously formed natural dams. According to Costa [1984] prerequisite conditions for most debris flows include an abundant source of unconsolidated fine-grained rock and soil debris, steep slopes, a large but intermittent source of moisture, and sparse vegetation. Several hypotheses have been formulated to explain mobilization of debris flows. Takahashi [1991] modelled the process as a water-saturated inertial grain flows governed by the *dispersive stress* concept of Bagnold. In this study we adopt Takahashi's theory as the most appropriate to describe the types of event observed in Switzerland.

The mechanism to disperse the materials in flow depends on the properties of the materials (*grain size*, *friction angle*), channel slope, flow rate and water depth, particle concentration, etc., and, consequently, the behavior of flow is also various.

2.2 Methods

Credal Sets and Probability Intervals. We restrict the attention to random variables which assume finitely many values (also called *discrete* or *categorical* variables). Denote by $\mathcal{X}$ the possibility space for a discrete variable X, with x a generic element of $\mathcal{X}$. Denote by $P(X)$ the mass function for X and by $P(x)$ the probability of $x \in \mathcal{X}$. Let a *credal set* be a closed convex set of probability mass functions. $\mathcal{P}_X$ denotes a generic credal set for X. For any event $\mathcal{X}' \subseteq \mathcal{X}$, let $\underline{P}(\mathcal{X}')$ and $\overline{P}(\mathcal{X}')$ be the *lower and upper probability* of $\mathcal{X}'$, respectively, defined by $\underline{P}(\mathcal{X}') = \min_{P \in \mathcal{P}_X} P(\mathcal{X}')$ and $\overline{P}(\mathcal{X}') = \max_{P \in \mathcal{P}_X} P(\mathcal{X}')$. Lower and upper (conditional) expectations are defined similarly. Note that a set of mass functions, its convex hull and its set of *vertices* (also called *extreme mass functions*) produce the same lower and upper expectations and probabilities.

Conditioning with credal sets is done by element-wise application of Bayes rule. The posterior credal set is the union of all posterior mass functions. Denote by $\mathcal{P}_X^y$ the set of mass functions $P(X|Y=y)$, for generic variables X and Y. We say that two variables are *strongly independent* when every vertex in $\mathcal{P}_{(X,Y)}$ satisfies stochastic independence of X and Y.

Let $\mathbb{I}_X = \{\mathbb{I}_x : \mathbb{I}_x = [l_x, u_x], 0 \leq l_x \leq u_x \leq 1, x \in \mathcal{X}\}$ be a set of probability intervals for X. The credal set originated by $\mathbb{I}_X$ is $\{P(X) : P(x) \in \mathbb{I}_x, x \in \mathcal{X}, \sum_{x \in \mathcal{X}} P(x) = 1\}$. $\mathbb{I}_X$ is said *reachable* or *coherent* if $u_{x'} + \sum_{x \in \mathcal{X}, x \neq x'} l_x \leq 1 \leq l_{x'} + \sum_{x \in \mathcal{X}, x \neq x'} u_x$, for all $x' \in \mathcal{X}$. $\mathbb{I}_X$ is coherent if and only if the related credal set is not empty and the intervals are tight, i.e. for each lower or upper bound in $\mathbb{I}_X$ there is a mass function in the credal set at which the bound is attained (see Campos et al. [1994]).

The Imprecise Dirichlet Model. We infer probability intervals from data by the *imprecise Dirichlet model*, a generalization of Bayesian learning from multinomial data based on soft modelling of prior ignorance. The interval estimate for value x of variable X is given by $[\#(x)/(N+s), (\#(x)+s)/(N+s)]$, where $\#(x)$ counts the number of units in the sample in which $X = x$, N is the total number of units, and s is a hyperparameter that expresses the degree of caution of inferences, usually chosen in the interval $[1, 2]$ (see Walley [1996a] for details). Note that sets of probability intervals obtained using the imprecise Dirichlet model are reachable.

Credal Networks. A credal network is a pair composed of a directed acyclic graph and a collection of conditional credal sets. A node in the graph is identified with a random variable X_i (we use the same symbol to denote them and we also use "node" and "variable" interchangeably). The graph codes strong dependencies by the so-called *strong Markov condition*: every variable is strongly independent of its nondescendant non-parents given its parents. A generic variable, or node of the graph, X_i holds the collection of credal sets $\mathcal{P}_{X_i}^{pa(X_i)}$, one for each possible joint state $pa(X_i)$ of its parents $Pa(X_i)$. We assume that the credal sets of the net are *separately specified* (Walley [1991]): this implies that selecting a mass function from a credal set does not influence the possible choices in others.

Denote by $\mathcal{P}$ the *strong extension* of a credal network. This is the convex hull of the set of joint mass functions $P(\boldsymbol{X}) = P(X_1, \ldots, X_t)$, over the t variables of the net, that factorize according to $P(x_1, \ldots, x_t) = \prod_{i=1}^{t} P(x_i | pa(X_i)) \quad \forall (x_1, \ldots, x_t) \in \times_{i=1}^{t} \mathcal{X}_i$. Here $pa(X_i)$ is the assignment to the parents of X_i

consistent with $(x_1, \dots, x_t)$; and the conditional mass functions $P(X_i \,|pa\,(X_i)\,)$ are chosen in all the possible ways from the respective credal sets. The strong Markov condition implies that a credal network is equivalent to its strong extension. Observe that the vertices of $\mathcal{P}$ are joint mass functions. Each of them can be identified with a Bayesian network (Pearl [1988]), which is a precise graphical model. In other words, a credal networks is equivalent to a set of Bayesian networks. This makes credal networks inherit some of the advantages of Bayesian nets, such as compactness of uncertainty representation and easy visualization, while presenting the additional characteristic to permit modelling based on weaker, and hence often more realistic, assumptions.

Computing with Credal Networks. We focus on the task called *updating*, i.e. the computation of $\underline{P}(X|E = e)$ and $\overline{P}(X|E = e)$. Here E is a vector of *evidence variables* of the network, in state e (the *evidence*), and X is any other node. The updating is intended to update prior to posterior beliefs about X. The updating can be computed by (i) exhaustively enumerating the vertices P_k of the strong extension; and by (ii) minimizing and maximizing $P_k(X|E = e)$ over k, where $P_k(X|E = e)$ can be computed by any updating algorithm for Bayesian networks (recall that each vertex of the strong extension is a Bayesian network).
The exhaustive approach can be adopted when the vertices of the strong extension are not too many. In general, non-exhaustive approaches must be applied as the updating problem is *NP-hard* with credal nets (Ferreira da Rocha and Cozman [2002]) also when the graph is a polytree. A polytree is a directed graph with the characteristic that forgetting the direction of arcs, the resulting graph has no cycles. In the present work the type of network, jointly with the way evidence is collected, make the exhaustive approach viable in reasonable times. Note that the exhaustive algorithm needs credal sets be specified via sets of vertices. We used the software tool *lrs* (http://cgm.cs.mcgill.ca/˜avis/C/lrs.html) to produce extreme mass functions from probability intervals.

3 The Credal Network

3.1 Causal Structure

The network in Fig. 1 expresses the causal relationships between the topographic and geological characteristics, and hydrological preconditions, already sketched in Sec. 2.1. The leaf node is the depth of debris likely to be transported downstream during a flood event. Such node represents an integral indicator of the hazard level.

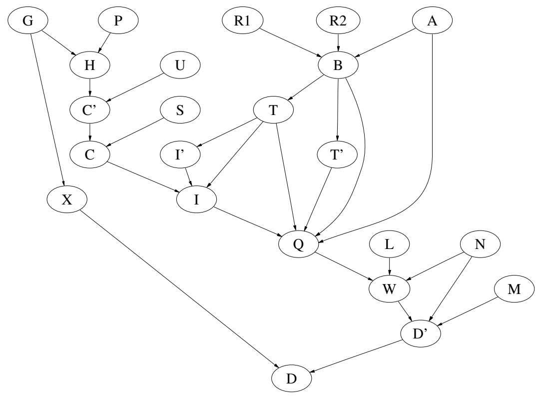

Figure 1: The causal structure.

In the following we describe the considerations that led to the network in Fig. 1. Node G represents the characteristics of the bedrock (*geology*) in a qualitative way. Debris flows require a minimum thickness of *colluvium* (loose, incoherent deposits at the foot of steep slope) for initiation, produced from a variety of bedrock. This is embedded in the graph with the connection to node X (*actual available debris thickness*) and expresses the propensity of different rock types to produce sediment. Additionally, bedrock properties influence the rate of infiltration and deep percolation, so affecting the generation of surface runoff and the concentration in the drainage network. This is accounted for by the connection of the geology to the *hydrologic soil type* (H), which influences the *maximum soil water capacity* (C').
The *soil permeability* (P), i.e. the rate at which fluid can flow through the pores of the soil, has to be further considered. If permeability is low, the rainfall will tend to accumulate on the surface or flow along the surface if it is not horizontal. The causal relation among geology and permeability determining the different hydrologic soil types was adopted according to Kuntner [2002]. The basic assumption is that soils with high permeability and extreme thickness show a high infiltration capacity, whereas shallow soils with extremely low permeability have a low infiltration capacity.
The *land use cover of the watershed* (U) is another significant cause of debris movement. It characterizes the uppermost layer of the soil system and has a definite bearing on infiltration.
We adopted the *curve number method* (USDA [1993]) to define the infiltration amount of the precipitation, i.e. the maximum soil water capacity. This method distinguishes hydrologic soil types which are supposed to show a particular hydrologic

behavior. For each land use type there is a corresponding curve number for each hydrologic soil type.
The amount of rainfall which cannot infiltrate is considered to accumulate into the drainage network (*surface runoff*), increasing the water depth and eventually triggering a debris flow in the river bed.
These processes are described by the deterministic part of the graph, related to runoff generation and Takahashi's theory, which takes into account topographic and morphologic parameters, such as *slope* (N) of the source area, *watershed morphology* (R_1 and R_2), *area* (A), *channel width* (L), and *precipitation intensity* (I').
The channel width is obviously decisive to determine the *water depth* (W), given the runoff generated within the watershed according to the standard hydraulic assumptions. Field experience in the study region indicates that debris flows often start in very steep and narrow creeks, with reduced accumulation area upstream.
The complexity and the organization of the channel geometry is therefore usually low and almost similar in the debris flow prone watersheds. For this reasons it was decided to adopt only three categories of channel width.
The climate of the regions in which debris flows are observed is as varied as geology and this was accounted for by defining several climatological regions, with different parameters of the depth-duration-frequency curve. In addition to the *duration* (T) and *effective rainfall intensity* (I) of a storm that ultimately produces a debris flow, the antecedent soil moisture conditions is recognized as an important characteristic. The significant period of antecedent rainfall varies from days to months, depending on local soil characteristics. According to the curve number theory, the transformation law to the *effective maximum soil water capacity* (C) depends only on the five-days antecedent rainfall amount corresponding to different *soil moisture conditions* (S).
We used the *linear theory of the hydrologic response* to calculate the *peak flow* (Q) values produced by constant-intensity hyetographs. We used the *multiscaling framework for intensity duration frequency curve* (Burlando and Rosso [1996]) coupled with the *instantaneous unit hydrograph* theory, proposed by Rigon et al. [2004]. Accordingly, the time to peak is greater than the rainfall duration and the *critical storm duration* (T') is independent of rainfall return period. The instantaneous unit hydrograph was obtained through the *geomorphological theory* (Rodriguez Iturbe and Valdes [1979]) and the *Nash cascade model of catchment's response*, where the required parameters (B_1 and B_2) were estimated from *Horton's order ratios* (R_1 and R_2), according to Rosso [1984].
By using the classical river hydraulics theory, the water depth in a channel with uniform flow and given discharge, water slope and roughness coefficient can be determined with the *Manning-Strickler formula* (see Maidment [1993]).
The *granulometry* (M), represented by the average particle diameter of the sediment layer, is required to apply Takahashi's theory. The *friction angle* was derived from the granulometry with an empirical one-to-one relationship. Takahashi's theory can finally be applied to determine the *theoretical thickness of debris* (D') that could be destabilized by intense rainfall events. The resulting value is compared with the *actual available debris thickness* (X) in the river bed. The minimum of these two values is the leaf node of the graph (D).

3.2 Quantification

Quantifying uncertainty means to specify the conditional mass functions $P(X_i|pa(X_i))$ for all the nodes X_i and the possible instances of the parents $pa(X_i)$. The specification is imprecise, in the sense that each value $P(x_i|pa(X_i))$ can lie in an interval. Intervals were inferred for the nodes G, P, U, N, H, and C', from the GEOSTAT database (Kilchenmann et al. [2001]) by the imprecise Dirichlet model (with s=2). The expert provided intervals for nodes L, M, R_1, R_2, and X. Functional relations between a node and its parents were available for the remaining nodes; in this case the intervals degenerate to a single 0-1 valued mass function. We detail the functional part in the rest of the section.
As mentioned in Sec. 2.1, the antecedent soil moisture conditions were accounted for by using the curve number method. The parametrization of the instantaneous unit hydrograph was obtained by using the number of theoretical linear reservoirs by which the basin is represented, $b_1 = 3.29 \cdot r_1^{0.78} \cdot r_2^{0.07}$; and by the time constant of each reservoir, $b_2 = .7 \cdot 0.251 \cdot (r_1 \cdot r_2)^{-.48} \cdot a^{0.38}$. Here b_1 depends on Horton's ratios, and b_2 is also function of the average travel time within the basin. For this we assumed the empirical expression reported by D'Odorico and Rigon [2003].
Given b_1 and b_2, following Rigon et al. [2004], we calculate the two characteristic durations, t and t', by solving the following system of two equations: $\alpha = [\frac{t}{b_2} \cdot (\frac{t'}{b_2})^{b_1-1} e^{-t'/b_2}]/[\gamma(b_1, \frac{t'}{b_2}) - \gamma(b_1, \frac{t'-t}{b_2})]$, and $\frac{t}{t'} = 1 - e^{-\frac{t}{b_2} \cdot \frac{1}{b_1-1}}$, where γ is the *incomplete lower gamma function* and α is a parameter, corresponding to the exponent of the *multiscaling*

intensity duration frequency curve.
We assume that these are in the form $i' = a(\tau_r) \cdot t^{-\alpha}$, where a is function of the return period τ_r of the event. To evaluate the effective intensity of rainfall, we have to impose the following transformation, taking account of the (effective) curve number, the corresponding dispersion term, and of the rainfall duration: $i = (i' \cdot t - \lambda(c)/10)^2/(i' \cdot t - \lambda(c)/10) + \lambda(c) \cdot 1/t$, where $\lambda(c) = 254 \cdot (100/c - 1)$ is the water depth absorbed by the soil of given curve number. The peakflow (Rigon et al. [2004]) can then be expressed as $q = a \cdot i/\alpha \cdot t/b_2 \cdot t'^{b_1-1} e^{-t'/b_2}$, and the corresponding waterdepth is $w = q/25\, l^{5/3} \sqrt{\tan n}$.
According to Takahashi [1991], we evaluate the debris thickness as $d' = w[k(\tan m'/\tan n - 1) - 1]^{-1}$. The relation is linear, with a coefficient taking into account the local slope n and the internal friction angle m' (which can be obtained from the granulometry m). $k = C_g(\delta_g - 1)$, with $\delta_g = 2.65$ the relative density of the grains, and $C_g \simeq 0.7$ the volumetric concentration of the sediments. The variables involved in the expression for d' must satisfy the constraints $1 + 1/k_2 \leq \tan m/\tan n \leq 1 + 1/k \cdot (1 + w/m)$. If the inequality on the left-hand side is violated, shallow landslides can occur also in absence of water depth, but technically speaking these are not debris flows. If the remaining inequality fails, the movable quantity is thinner than the granulometry and no flow can be observed.
d' is a theoretical value for the movable quantity, which does not take into account how much material is physically available. As the actual movable quantity cannot exceed the available material x, the final relation is given by $d = \min\{x, d'\}$.

4 Case Studies

We validate the model in preliminary way by an empirical study involving six areas of the Ticino canton. The network was initially fed with the information about the areas reported in Tab. 1, the estimated rainfall intensity on them for a return period of 10 years, and the geomorphological characteristics of the watershed. The estimated rainfall intensity is the expected frequency level of precipitations in a certain region during a future period. Using the estimated rainfall intensity allowed us to re-create the state of information existing 10 years ago[3] about precipitations in the areas under consideration. This is a way to check whether the network would have been a valuable tool to prevent the debris flows that actually happened in the six areas. The results of the analysis are in Tab. 2. We use the probabilities of defined debris thickness to be transported downstream as an integral indicator of the hazard level.

[3]The number of years is an arbitrary choice.

Table 1: Details about the case studies. (Note that P is not available. This is the common case as evaluation of permeability presents technical difficulties.)

Node	Cases					
	1	2	3	4	5	6
G	Gneiss	Porphyry	Limestone	Gneiss	Gneiss	Gneiss
A	0.26	0.32	0.06	0.11	0.38	2.81
M	10–100	≤10	≤10	100–150	≤10	150–250
U	Forest	Forest	Forest	Vegetation	Forest	Bare soil
N	20.8	19.3	19.3	21.8	16.7	16.7
L	4	6	4	8	4	8
R_1	0.9	0.6	0.7	0.9	0.9	0.8
R_2	1.5	3.5	3.5	3.5	2.3	2.1

Table 2: Posterior probabilities of node D, i.e. of the movable debris thickness (in centimeters). The probabilities are displayed by intervals in case 2.

Thickness	Cases					
	1	2	3	4	5	6
<10	0.011	[0.084,0.087]	0.083	0.196	0.087	0.005
10–50	0.048	[0.263,0.273]	0.275	0.388	0.139	0.013
>50	0.941	[0.639,0.652]	0.642	0.416	0.774	0.982

In cases 1 and 6 the evidences are the most extreme out of the six cases and indicate a high debris flow hazard level, corresponding to an instable debris thickness greater than 50 cm. In case 6 the relatively high upstream area (2.81 km^2), large channel depth, and the land cover (bare soil, low infiltration capacity) explain the results. In case 1 the slope of the source area (20.8°) plays probably the key role. In cases 2 and 3 the model presents a non-negligible probability of medium movable debris thickness. Intermediate results were obtained for case 5 due to the gentler bed slope (16.7°) as compared with the other cases. In case 4 the hazard probability is more uniformly distributed, and can plausibly be explained with the very small watershed area and the regional climate, which is characterized by low small rainfall intensity as compared with other regions.
We simulated also the historical events, by instantiating (as opposed to using the estimated rainfall intensity) the actual measured rainfall depth, its duration and the antecedent soil moisture conditions. Also in this setup the network produced high probabilities of significant movable thickness. (The probabilities are not reported for lack of space.)
As more general comment, it is interesting to observe that in almost all cases the posterior probabilities are nearly precise. This depends on the strength of the evidence given as input to the network about the cases, and by the fact that the flow process can partially be (and actually is) modelled functionally.

Now we want to model the evidence in even more realistic way with respect to the grain size of debris material. Indeed, granulometry is typically known only partially, and this limits the real application of physical theories, also considered that granulometry is very important to determine the hazard.

We model the fact than the observer may not be able to distinguish different granulometries. To this extent we add a new node to the net, say O_M, that becomes parent of M. O_M represents the observation of M. There are five possible granulometries, m_1 to m_5. We define the possibility space for O_M as the power set of $\mathcal{M} = \{m_1, \ldots, m_5\}$, with elements $o_{\mathcal{M}'}$, $\mathcal{M}' \subseteq \mathcal{M}$. The observation of granulometry is set to $o_{\mathcal{M}'}$ when the elements of $\mathcal{M}'$ cannot be distinguished. $P(m|o_{\mathcal{M}'})$ is defined as follows: it is set to zero for all states $m \in \mathcal{M}$ so that $m \notin \mathcal{M}'$; and for all the others it is *vacuous*, i.e. the interval $[0, 1]$ (the intervals defined this way must then be made reachable). This expresses the fact that we know that $m \in \mathcal{M}'$, and nothing else.

Let us focus on case 6 for which the observation of grain size is actually uncertain. From the historical event report, we can exclude that node M was in state m_1 or m_2. We cannot exclude that m_4 was the actual state (m_4 is the evidence used in the preceding experiments), but this cannot definitely be established. We take the conservative position of letting the states m_3, m_4 and m_5 be all plausible evidences by setting $O_M = o_{\{m_3,m_4,m_5\}}$. The interval probabilities become $[0.002, 0.008]$, $[0.010, 0.043]$, and $[0.949, 0.988]$, for debris thicknesses less than 10, in the range 10–50, and greater than 50, respectively. We conclude that the probability of the latter event is very high, in robust way with respect to the partial observation of grain size.

5 CONCLUSIONS

We have presented a model for determining the hazard of debris flows based on credal networks. The model unifies human expertise and quantitative knowledge in a coherent framework. This overcomes a major limitation of preceding approaches, and is a basis to obtain credible predictions, as shown by the experiments. Credible predictions are also favored by the soft-modelling made available by imprecise probability through credal nets.

The model was developed for the Ticino canton, in Switzerland. Extension to other areas is possible by re-estimating the probabilistic information inferred from data, which has local nature.

Debris flows are a serious problem, and developing formal models can greatly help us avoiding their serious consequences. The encouraging evidence provided in this paper makes credal networks be models of debris flows worthy of further investigation.

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